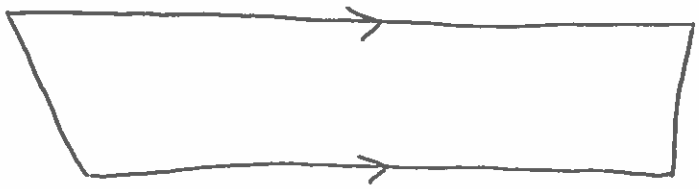
Solutions

1.1: Formal Logic
Logical Equivalences and Examples

Definition 1. Two statements are logically equivalent if they have the same T/F values for all cases; i.e. if they have the same truth tables. ~~If two statements, A and B, are logically equivalent, then we write $A \sim B$.~~

Example 1. Consider the following high school geometry theorem.

If a quadrilateral has a pair of parallel sides, then it has a pair of supplementary angles.



This statement is of the form $p \to q$ where

P = "the quadrilateral has a pair of parallel sides" and

q = "the quadrilateral has a pair of supplementary angles."

Consider the statement

"If a quadrilateral does not have a pair of supplementary angles, then it does not have a pair of parallel sides."

This statement is of the form $\neg q \to \neg p$.

It must be true because $p \to q$ is logically equivalent to $\neg q \to \neg p$.

Definitions 2-3. Not only is the truth table for an implication statement $(p \to q)$ the most unintuitive, but there are also many ways to rearrange it. For instance, by negating both and flipping the order we get the contrapositive $(\neg q \to \neg p)$, which is logically equivalent to $(p \to q)$. On the other hand, by simply flipping the order we get the converse $(q \to p)$, which is not logically equivalent.

P	q	$P \to q$
T	T	T
T	F	F
F	T	T
F	F	T

P	q	$\neg q$	$\neg P$	$\neg q \to \neg P$
T	T	F	F	T
T	F	T	F	F
F	T	F	T	T
F	F	T	T	T

P	q	$q \to P$
T	T	T
T	F	T
F	T	F
F	F	T

Exercise 1. If Aaron is late, then Bill is late, and. if both are late, then class is boring. Suppose that class is not boring. What can you conclude about Aaron?

Rox #	p	q	r	$p \to q$	$p \wedge q$	$(p \wedge q) \to r$	S
1.	T	T	T	T	T	T	T
2.	T	T	F	T	T	F	F
3.	T	F	T	F	F	T	F
4.	T	F	F	F	F	T	F
5.	F	T	T	T	F	T	T
6.	F	T	F	T	F	T	T
7.	F	F	T	T	F	T	T
8.	F	F	F	T	F	T	T

P="Aaron is late."
q="Bill is late."
r="Class is boring."

$S = (p \to q) \wedge [(p \wedge q) \to r]$

It is given that S is true. So ignore rows 2-4. We suppose r is false, so also ignore ~~Rows~~ 1, 5 and. Remaining rows are 6 and 8. In either case, p is false and so Aaron is not late.

Exercise 2. Let the following statements be given.

p = "You can vote."
q = "You are under 18 years old."
r = "You are from Mars."

(a) Translate the following statement into symbols of formal logic.

You can't vote if you are under 18 years old or your are from Mars.

(b) Give the contrapositive of this statement in the symbols of formal logic.

(c) Give the contrapositive in English.

(a) $(q \vee r) \to \neg p$ (b) $p \to \neg(q \vee r)$

(c) If you can vote, then you are not, under 18 years old or from Mars.
If you can vote, then you are not under 18 years old and not from Mars.

DeMorgan's Laws. (See De Morgan's Laws)

$\neg(p \vee q)$ is logically equivalent to $\neg p \wedge \neg q$
$\neg(p \wedge q)$ is logically equivalent to $\neg p \vee \neg q$

~~P~~ This is Exercise 10 in the Practice problems.

Homework. (Due Sept 3, 2018) Section 1.1: 4, 6

Practice Problems. 5, 7-11, 13-14, 18-21, **22-30**